

**UNCLASSIFIED**

---

**AD 402 194**

*Reproduced  
by the*

**DEFENSE DOCUMENTATION CENTER**

**FOR**

**SCIENTIFIC AND TECHNICAL INFORMATION**

**CAMERON STATION, ALEXANDRIA, VIRGINIA**



---

**UNCLASSIFIED**

TRG-140-FR

Submitted by:  
TRG, Incorporated  
2 Aerial Way  
Syosset, New York

REPORT ON A SEMINAR ON THE  
HYDRODYNAMIC THEORY ASSOCIATED  
WITH SHIP MOTION IN WAVES

Authors: Paul Kaplan  
Paul Kaplan  
Jack Kotik  
Jack Kotik

Approved: Jack Kotik

Submitted to:  
Office of Naval Research  
Washington, D.C.

## TABLE OF CONTENTS

	<u>Page No.</u>
1. INTRODUCTION -----	1
2. SUMMARY OF DISCUSSIONS -----	4
2.1 Present Analytical Methods for Predicting the Motions of Ships in Waves -----	4
2.2 Present Methods of Determining the Damping and Inertia Coefficients in the Equations of a Ship in Regular (Monodirectional, Monochromatic) Waves--	5
2.2.1 Two-Dimensional Techniques -----	5
2.2.2 Three-Dimensional Methods, Zero Speed -----	9
2.2.3 Three-Dimensional Methods, Forward Speed ---	9
2.3 Exciting Forces on Ships in Waves -----	13
2.4 Problems and Recommendations -----	15
2.4.1 Consistent and Inconsistent Linearizations--	15
2.4.2 Relations Between Cross-Coefficients of Inertia and Damping -----	16
2.4.3 Relations of the Kramers-Kronig Type Between Damping and Inertia Coefficients----	17
2.4.4 Effect of Viscosity in Wave Problems -----	17
2.4.5 Time-Domain Formulation of the Equations of Motion -----	18
2.4.6 Recommendations -----	18
References -----	19

REPORT ON A SEMINAR ON THE HYDRODYNAMIC THEORY ASSOCIATED  
WITH SHIP MOTION IN WAVES

by

Paul Kaplan and Jack Kotik

1. INTRODUCTION

The theory of the motions of a ship in waves was developed along two major diverse paths. Various practical investigators and some designers have used results based on many questionable assumptions, while more academic investigators have studied isolated parts of an overall motion analysis (e.g. damping and added mass coefficients) or have treated idealized problems having no direct application to practice. As an example of practical approaches the work of Korvin-Kroukovsky and Jacobs<sup>[1]</sup> develops a set of coupled linear differential equations for heave and pitch of a ship in regular head seas, and the coefficients used there are the values obtained by "strip theory", i.e. by use of integrals of two-dimensional quantities. A number of different methods have been used to calculate the two-dimensional coefficients which appear in such sets of differential equations, each having different degrees of validity and simplicity. Similarly there exists a number of different expressions for hydrodynamic forces on oscillating bodies advancing on or below the free surface, which have been obtained within what is considered to be a three-dimensional analysis (subject to certain simplifications such as thin-ship, slender-body, symmetry, etc.)

It seemed appropriate to bring together a group of investigators who were active in the various theoretical aspects of the problem, and who represented some of the different approaches to the problems presently being investigated, in order to conduct an informal seminar on this subject. The participants in the seminar and their affiliations are listed below:

Dr. Klaus Eggers, Institut für Schiffbau, Hamburg University  
Dr. Paul Kaplan, TRG, Inc.\*  
Dr. Samuel Karp, New York University; consultant to TRG, Inc.  
Dr. Jack Kotik, TRG, Inc.  
Dr. Richard C. MacCamy, Carnegie Inst. of Tech.  
Dr. J. Nicholas Newman, David Taylor Model Basin  
Dr. Reiner Timman, Technological University, Delft

These scientists were, in the main, primarily concerned with problems related to the theoretical calculation of various hydrodynamic coefficients used in the equations of motion, rather than with the complete formulation of equations for calculation of actual motions. While certain of the participants had experience in the latter area, the majority interest led to concentration on the more theoretical hydrodynamic aspects of the ship motion problem. However, some attention was paid to the formulation of different types of equations of motion, and other aspects closer to practical utility. Nevertheless, it still remains to integrate the results of this seminar, and work that was produced or will be produced as a result of its recommendations, into more effective methods for determining the motion of a ship in waves.

The general topics covered were roughly as indicated below:

- a) Damping and inertia coefficients for the forced periodic motion of a body at zero speed.
- b) Influence of forward speed on the above.
- c) Cross-coupling terms between different degrees of freedom.
- d) Evaluation of forcing functions due to waves incident on a moving ship.
- e) Equations of motion, including convolution-type integro-differential equations.

---

\* Currently at Oceanics, Inc.

The main emphasis was on heave and pitch motions, but some consideration was given to rolling and the general problem of lateral motions. No attempt was made, either in the seminar or in this report, to arrive at an encyclopedic summary of past work and the present state of knowledge\*. The participants felt free to devote their attention to problems which most interested them, and as a result some important questions were considered briefly or not at all. In the case of those problems which were discussed at length considerable attention was paid to the practical utility of the existing and anticipated results.

This seminar was conducted under the sponsorship of the Office of Naval Research under Contract Nonr-3175(00).

---

\* Vossers<sup>[0]</sup> has published an extensive summary of the subject which can be recommended as a starting point.

## 2. SUMMARY OF DISCUSSIONS

### 2.1 Present Analytical Methods for Predicting the Motions of Ships in Waves

A simple method of analysis for the problem of a ship in regular waves is to assume that it is a linear uncoupled dynamic system in its various degrees of freedom. Since such equations are easily solved the main problem is to determine the appropriate coefficients, including the oscillatory forcing functions, from available hydrodynamic theory. An example of this approach is the paper by Weinblum and St. Denis<sup>[2]</sup> where the hydrodynamic techniques used were the Froude-Kryloff hypothesis (the assumption that the forces due to waves are the same as those exerted on the water displaced by the ship, i.e. neglecting interactions between fluid and ship), two-dimensional damping and added masses for ship sections which were integrated over the ship length in accordance with strip theory, and the use of infinite-fluid finite-length inertia factors for three-dimensional corrections. No comparisons with experiment were made for any practical ship form in [2].

A more complicated set of linear equations, including coupling terms, showing fair agreement with experiment for five ship forms, was given by Korvin-Kroukovsky and Jacobs<sup>[1]</sup>. The technique used there is appealing, but the evaluation of the various hydrodynamic coefficients used in the equations is still the subject of much discussion and research. In addition, certain questions arise as to the adequacy of the equations themselves when considering non-monochromatic waves as well as problems of nonlinearity. While these equations have been successful in certain cases, and of course only for problems of heave and pitch in monochromatic head seas, the need for appropriate tools with wider applicability with regard to ship form and number of degrees of freedom is certainly evident. However, it is desirable to extend their equations to six degrees of freedom and monochromatic waves of arbitrary (single) heading, and to obtain more accurate coefficients. This accounts for the great interest at the seminar in the evaluation of damping and inertia coefficients, coupling terms, forcing functions, etc.

## 2.2 Present Methods of Determining the Damping and Inertia Coefficients in the Equations of Motion of a Ship in Regular (monodirectional, monochromatic) Waves.

### 2.2.1 Two-Dimensional Techniques

Considerable effort has been devoted to the two-dimensional problem of the forces acting on a section oscillating at the free surface. The results are expressions for the damping coefficient (or wave amplitude coefficient) and the added mass coefficient of the section, as functions of a dimensionless frequency parameter. The various techniques used to determine these quantities are described in the following brief outline.

A simple method used for determining the damping due to heaving motion of a section on the free surface is due to Havelock and Holstein,<sup>[3],[4]</sup> and makes use of a distribution of sources along the body surface, with the strength proportional to the normal velocity along the body contour. The radiated wave amplitude is found and this leads to a value of the damping coefficient. A more useful and also more valid method of obtaining both sectional damping and added mass is that of Grim<sup>[5]</sup>, which is applied to a mathematical family of sections known as the Lewis forms. Grim satisfies the boundary condition on the body at a finite number of points. He has published the results for a variety of sections. Information for other sections can be obtained by interpolation from Grim's charts.

One of the first rigorous treatments of a linearized forced motion problem was Ursell's treatment<sup>[6]</sup> of the heaving semi-immersed circular cylinder. The problem was transformed to that of solving an infinite linear system. Convergence was proved, and the wave amplitude was computed numerically by solving a truncated set of linear equations. More recently Porter<sup>[7]</sup> has extended Ursell's method. He transformed the problem of a heaving section to that of solving an infinite linear system whose coefficients involve the coefficients in the series expansion of the conformal mapping of the exterior of the section on the exterior of a circular section. He



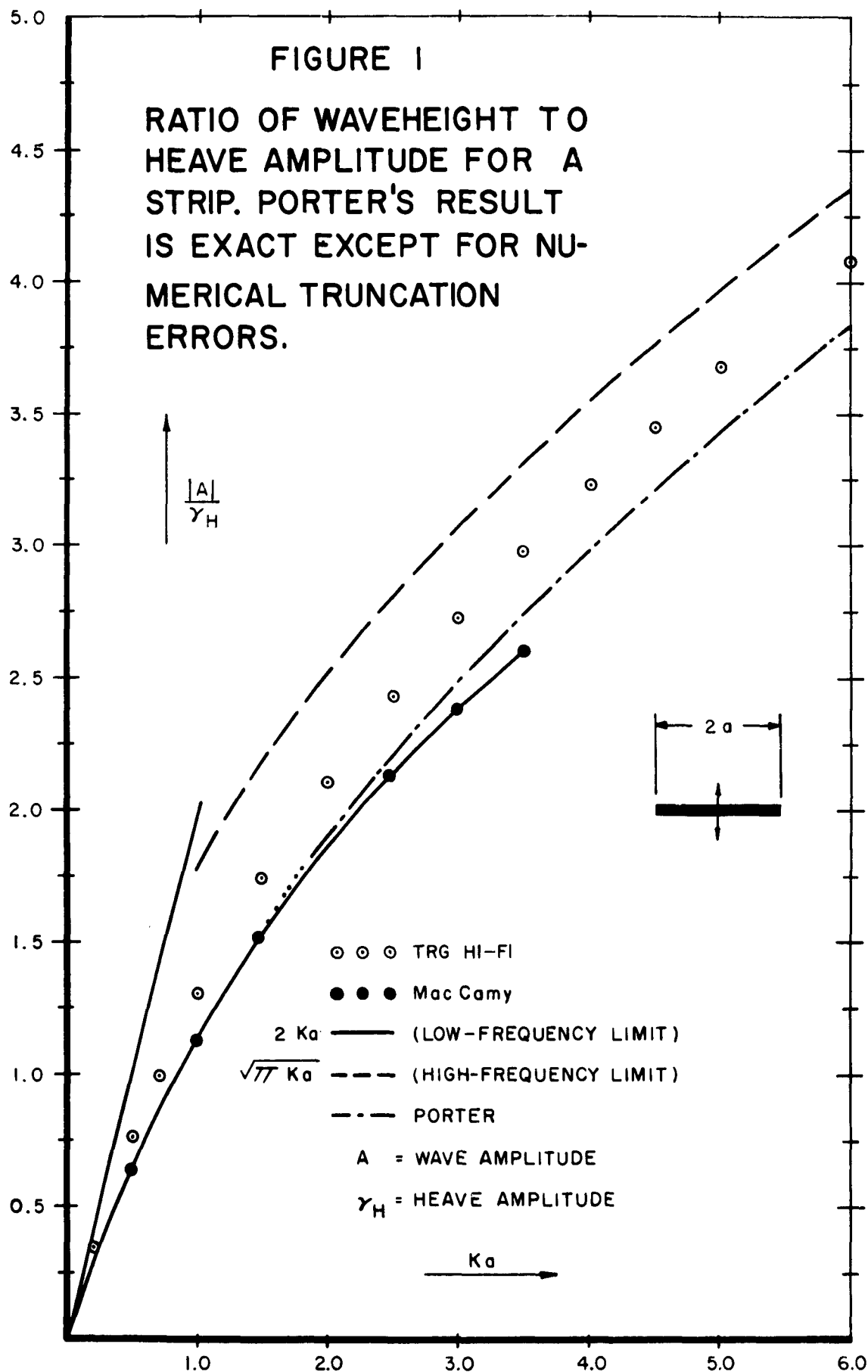
programmed the IBM 704 computer to solve truncated sets of linear equations, and computes wave amplitude, virtual mass and the pressure on the section at discrete points. The number of equations treated is an input variable and by increasing the number until the results converge one can be assured of a quite accurate solution. In the case of a circle the agreement with Ursell is excellent; which is to be construed as a verification of Ursell's results. Results for a number of other sections are presented in [7], and the computer program can be used for additional cases as required. In conjunction with a program for determining conformal mappings numerically, the Porter program could be used to treat arbitrary sections.

Kaplan and Jacobs<sup>[8]</sup> computed added-mass and damping coefficients for sections by a perturbation method assuming small beam. While the work does show good agreement with known results such as those for a circular or elliptical section, it is known that there is an error of the order of the vertical added mass coefficient in the asymptotic value at large frequencies.

Tasai<sup>[9]</sup> has computed virtual mass and damping for a number of simple forms by a method which appears to resemble Porter's except that the calculations were not mechanized. His results are generally very close to Porter's, although there are some deviations.

MacCamy<sup>[10]</sup> has developed a perturbation theory for sections of small draft. The integral equation to which he reduced the problem has been treated numerically for a heaving strip. A third solution was obtained by Porter (see Figure 1.) In addition, MacCamy has investigated related problems such as the behaviour of the potential at the confluence of two different boundary conditions, and the behaviour of the potential at low frequencies. To appreciate the second problem note that the problem at zero frequency is a Neumann problem whose solution is not unique. Yet for every non-zero frequency the potential is uniquely determined. To which of the many solutions of the zero-frequency problem does the potential tend as the frequency tends toward zero? MacCamy has sketched a proof that

$$\phi = \phi_0 + (\alpha + 4\log K) \int V_n ds + O(K \log K) \text{ as } k \rightarrow 0, \text{ where}$$



$$K = \sigma^2/g = 2\pi/\lambda$$

$\phi_0$  = any zero-frequency potential

$\alpha$  = a constant that depends on the choice of  $\phi_0$

$V_n$  = prescribed normal velocity on the section.

For a symmetrical motion (heave) of a symmetrical section  $\int V_n ds \neq 0$  in general so that an "infinite constant" appears. This accounts for the logarithmic infinity in the virtual mass of a heaving section at zero frequency, but it does not affect the leading term in the damping coefficient at low frequency.

The asymptotic behaviour of the wave amplitude and added-mass coefficients at low frequency has been investigated by a number of authors with varying degrees of rigour. It has been known for some time that the wave-amplitude coefficient for the heaving motion of an arbitrary section is

$$2Ka,$$

where  $2a$  is the width at the waterline. Ursell<sup>[11]</sup> has shown how to find the leading term in the wave amplitude for the heaving and rolling motion of sections for which the mapping function is known. He also found the leading term of the added-mass coefficient for heave at low frequency.

The high-frequency asymptotics has been investigated by Ursell by transforming the usual integral equation of the problem into another integral equation whose kernel is small at high frequency. The method was applied to a heaving circular cylinder and a heaving sphere, with the following results:

Cylinder:

$$\text{Wave amplitude coefficient: } \frac{4}{Ka}$$

$$\text{Added-Mass coefficient: } 1 - \frac{4}{3\pi Ka}$$

TRG has proposed a simple formula for the leading term of the wave-amplitude coefficient at high frequency in terms of the local behaviour

of the infinite-frequency potential at the intersection of the section and the free surface. The method has been applied to the following cases:

Elliptic Cylinder: Roll and Heave

Horizontal Strip: Roll and Heave

Vertical Strip: Roll

Lenticular Cylinder: Heave

The results agree with Ursell in all cases for which comparison is possible.

A substantial forward step is the introduction of the Hi-Fi method for heave by TRG. This method uses the infinite-frequency potential to produce a simple approximation to the wave-amplitude coefficient which is asymptotically valid both at low and high frequencies. Numerical calculations in a number of cases show that the error is small at all frequencies. (See Figures 2,3).

As far as experimental work is concerned, very little of a fundamental nature has been done directly on the two-dimensional problem. Stelson has measured the virtual mass of a heaving sphere<sup>[13]</sup>. An earlier set of investigations of both virtual mass and damping of two-dimensional forms was carried out by Holstein<sup>[12]</sup> but there was a large amount of scatter in the data so that comparisons of theory and experiment were not very definitive. What appears to be the most useful experimental data concerning two-dimensional virtual mass and damping is that obtained by Porter<sup>[7]</sup>. The total hydrodynamic force on an oscillating semicircular cylinder was obtained and compared with theory, showing excellent agreement. While no separation of the total force into virtual mass and damping components was possible in those tests due to noise and other instrumentation imperfections, it appears that this result adds support to the theoretical results. Nevertheless, a precise measurement and separation of the virtual mass and damping effects for such a fundamental form as a semicircle would serve as a basic reference for comparison with theoretical results, and it is recommended that this be carried out\*.

---

\* Our attention has been called to the work of Yun-Sheng Yu, Hydrodynamics Laboratory, M.I.T., who compared theory and experiment (wave amplitude and added-mass) for a heaving semi-circle.

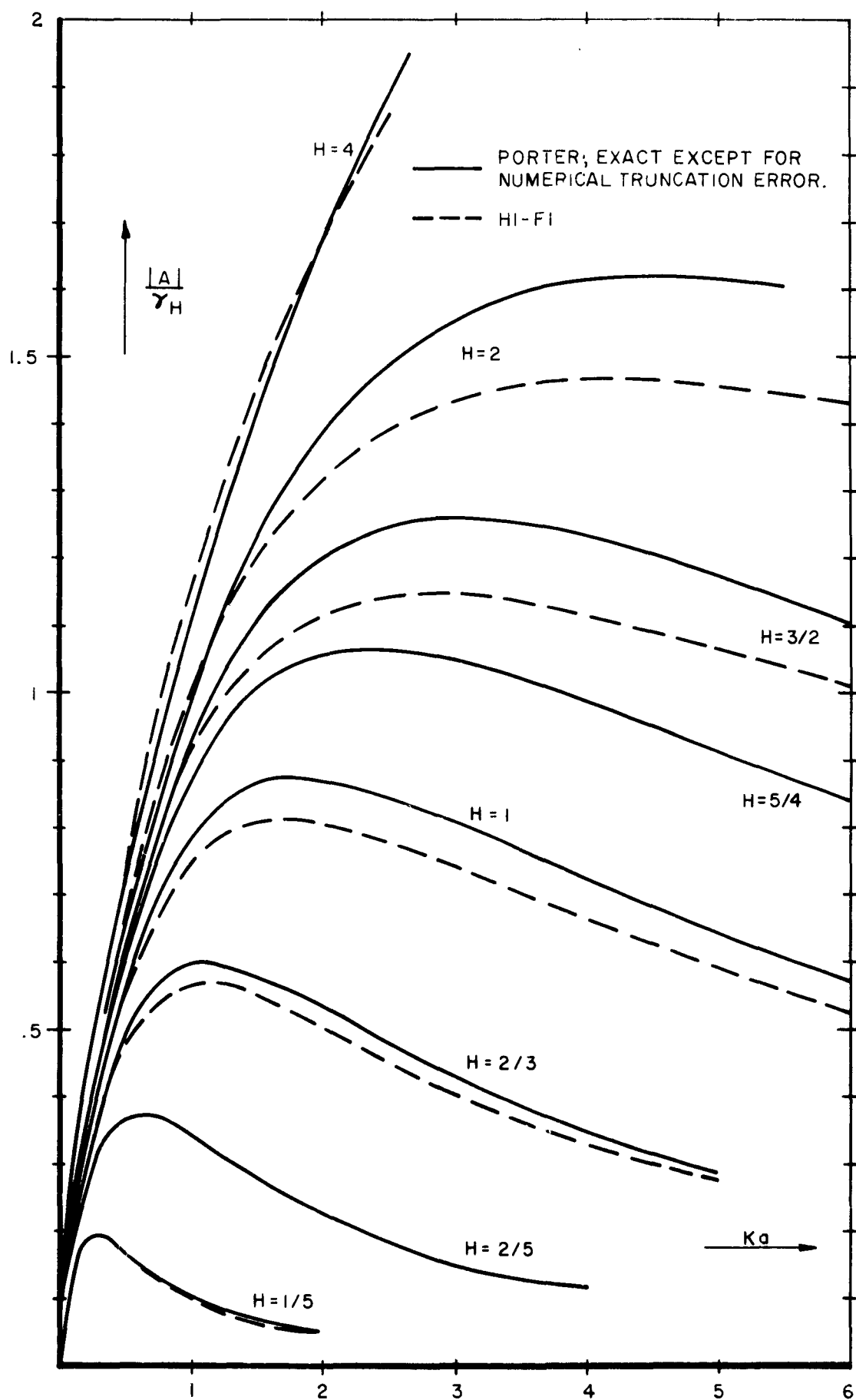


FIGURE 2

RATIO OF WAVEHEIGHT TO HEAVE AMPLITUDE  
FOR ELLIPTIC CYLINDERS.  $H = \text{HALF - BEAM / DRAFT}$ .

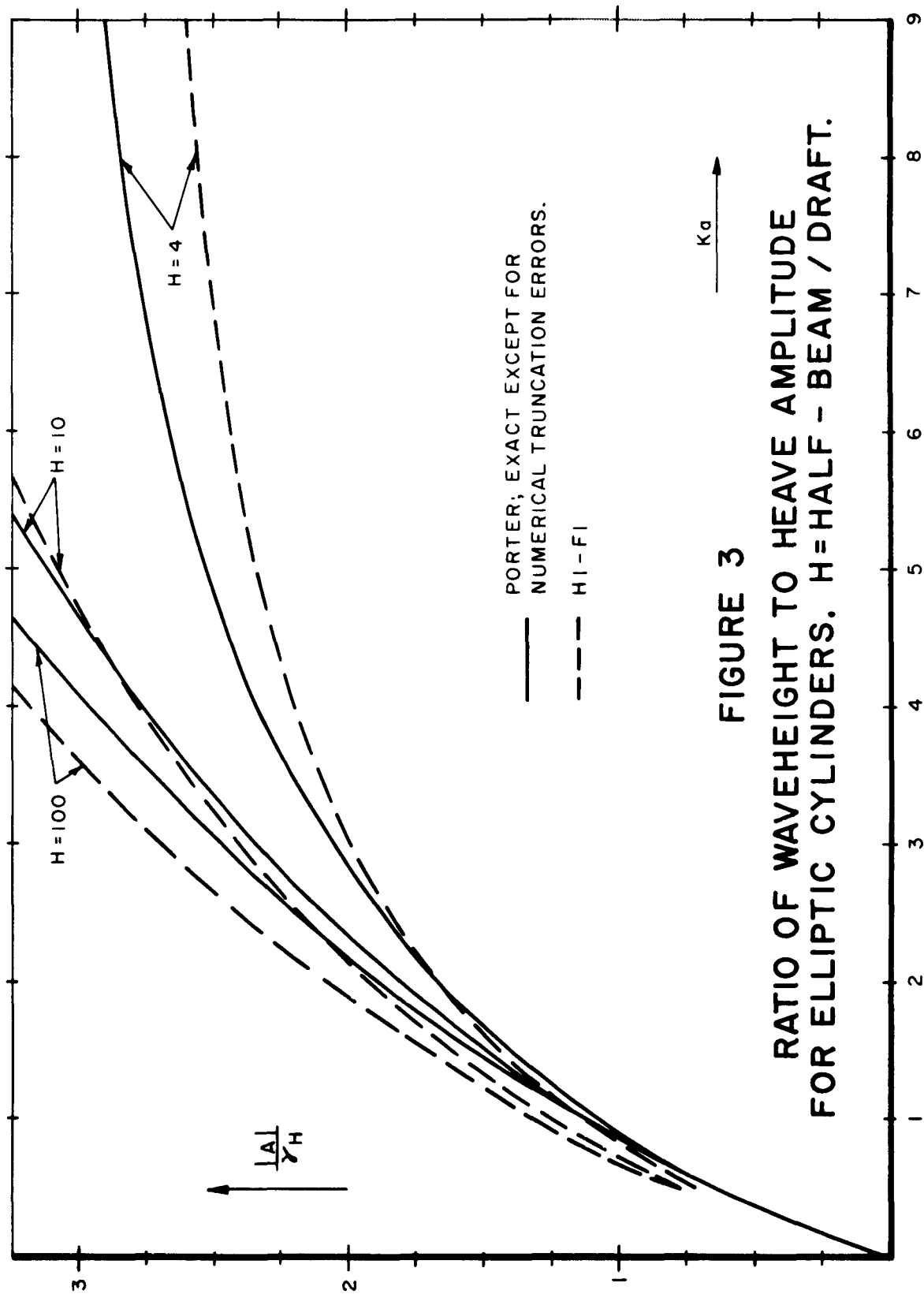


FIGURE 3  
RATIO OF WAVEHEIGHT TO HEAVE AMPLITUDE  
FOR ELLIPTIC CYLINDERS.  $H = \text{HALF - BEAM / DRAFT}$ .

### 2.2.2 Three-Dimensional Methods, Zero Speed

The heaving motion of a semi-immersed sphere has been treated by Havelock<sup>[14]</sup>, Ursell<sup>[15]</sup>, Barakat<sup>[16]</sup>, TRG<sup>[17]</sup> and Porter<sup>[18]</sup>. Havelock formulated the problem as an infinite linear system, which he truncated and solved numerically. Barakat did the same and attempted to perform the numerical work with greater accuracy and for a larger frequency range. Although his results are similar to Havelock's there are discrepancies, and there is conclusive evidence in Havelock's favor. Ursell found the high-frequency behaviour using the method he first applied to the circular cylinder, and found

$$h(\text{Havelock's damping parameter}) = \frac{27}{4(Ka)^4}$$

$$\text{added-mass coefficient} = \frac{1}{2} - \frac{3}{16Ka}$$

Porter has very recently obtained numerical results which are in excellent agreement with Havelock. Using the three-dimensional form of the Hi-Fi method TRG has calculated Havelock's damping parameter. The results are shown in the Figure 4.

MacCamy has treated the heaving circular disc by solving an integral equation<sup>[10]</sup>. He finds the wave amplitude and the pressure distribution on the disc. He can then compute the damping coefficient in two different ways, and the results are not in complete agreement. TRG has computed the damping coefficient for a heaving circular disc by the Hi-Fi method. The results are compared in Figure 5.

### 2.2.3 Three-Dimensional Methods, Forward Speed

The problem of an oscillating surface ship at zero speed is easily linearized by assuming either a thin ship or small motions. The situation with forward speed is more complicated. In the method of Stoker and Peters, [19], who perturb the nonlinear problem for small beam, various interesting quantities are zero to first order. On the other hand a perturbation around zero speed for a ship having finite dimensions has not been reported in the literature.

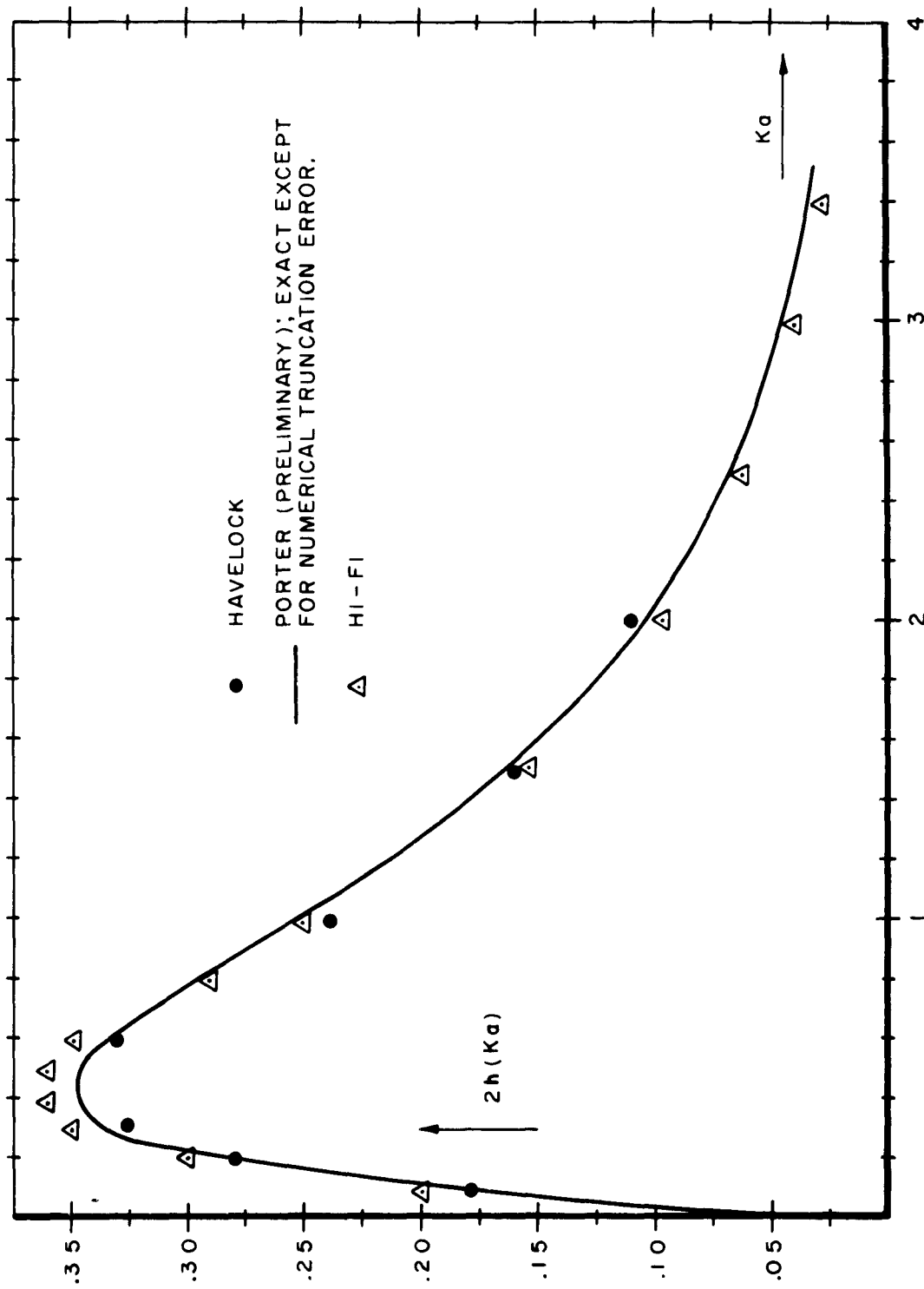


FIGURE 4  
HAVELOCK'S DAMPING PARAMETER  
FOR A HEAVING SPHERE



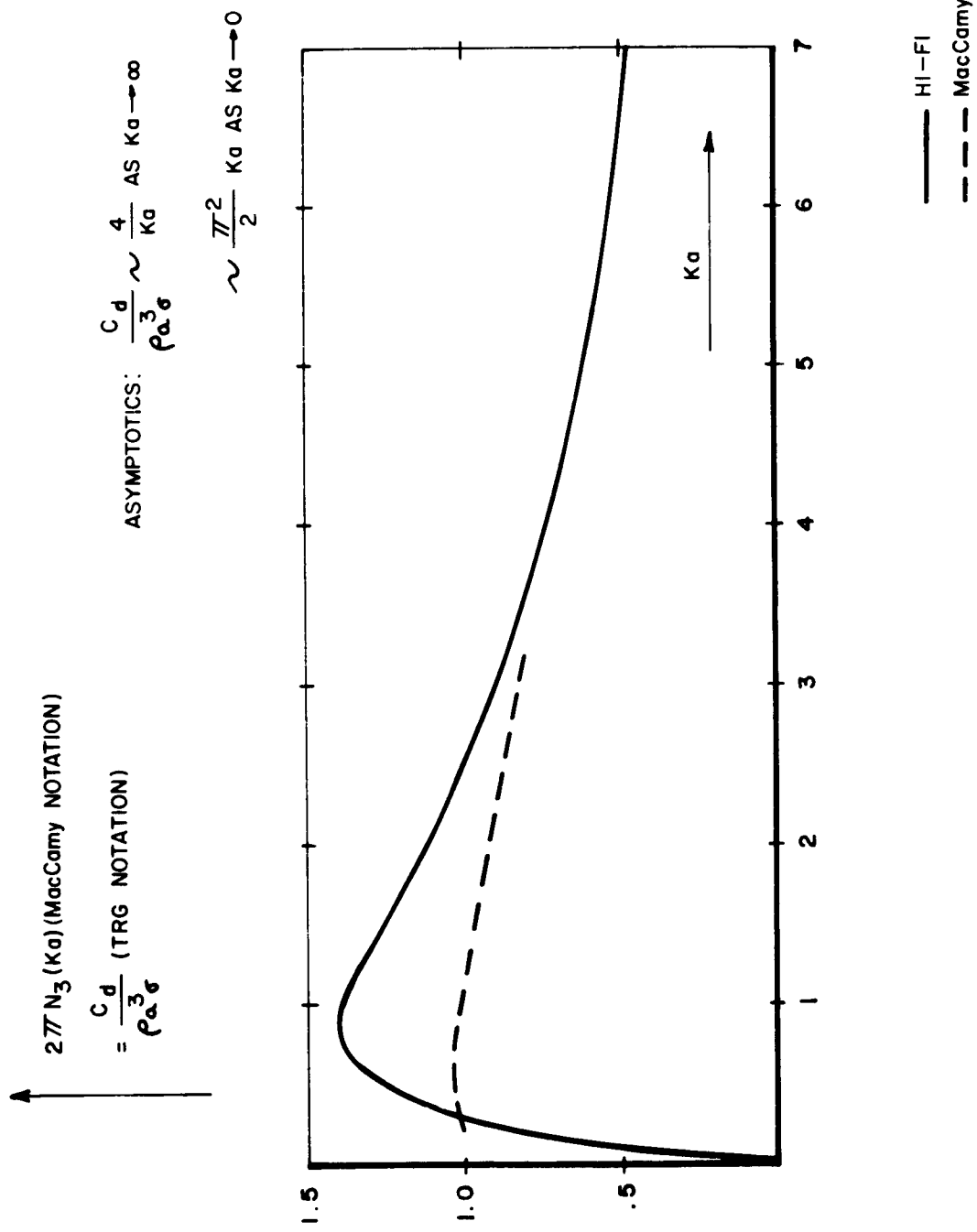


FIGURE 5  
DAMPING COEFFICIENT FOR A HEAVING CIRCULAR DISK

For a submerged body linearization of the free surface condition has been justified on the basis of sufficient distance of the body from the free surface, together with small motions. However, although useful practical results have been obtained in this manner the nature of the underlying perturbation theory, if any, is far from clear.

Newman<sup>[20]</sup> has carried out a perturbation expansion in which the amplitude of the incident waves, the amplitude of ship oscillation and the ship's beam are taken as independent small parameters. The calculation is pushed far enough to yield complicated but nonvanishing expressions for all the interesting quantities.

A technique developed by Eggers<sup>[37]</sup> gives the damping and also added resistance due to oscillatory motions of sources and/or dipoles moving with forward speed, below the free surface, by means of an energy analysis. The problem for any practical application is to determine the necessary distribution strengths and to carry out the required integrations. It is possible that Eggers' work will lead to a useful method of computing the damping of submerged bodies.

More recently Vossers<sup>[21]</sup> has published a slender-body theory (beam and draft of equal small order compared to length) in which the linearized free surface condition is assumed ab initio.

The discussions at the symposium, on this subject, were not extensive and have been superseded by subsequent work.

As far as experimental results are concerned, a number of investigations using oscillators and related techniques have been carried out in order to determine the virtual mass and/or damping of various ship hulls. Haskind and Riman<sup>[38]</sup> carried out an experimental study which resulted in values of virtual mass and damping of a simple ship form based on an analytic representation as a member of a simple family of ships. The results obtained therein appeared to agree quite closely with Haskind's earlier theory, but there is insufficient

information on the exact data as well as the possible existence of experimental errors which are not considered in detail in that study. A later set of experiments on a Series 60 hull was carried out by Gerritsma<sup>[39a, 39b]</sup>, where comparisons were made with the results of two-dimensional theories such as Havelock-Holstein and Grim, which were integrated over the hull in a strip theory application. Better agreement was obtained with Grim's results, but even there the departure in heave damping was quite large. Experimental measurements on a mathematical ship form model were carried out with an oscillator by Golovato<sup>[40]</sup> and expressions for all of the hydrodynamic forces and moments due to heave and pitch were determined. Newman<sup>[41]</sup> made comparisons of his theoretical results for damping with these experimental data and the results showed large differences, with the theoretical results much higher than the experiments. While some experimental error may be present, the order of the differences indicates that a fundamental difference is evident. Since thin-ship theory is supposed to be valid only for thin ships, a model of a thin ship, mathematically defined for simplicity, was oscillated by Gerritsma<sup>[42]</sup> to obtain data for comparison with Newman's theory. The results indicated experimental magnitudes of damping larger than theory, and led to speculation about the relative importance of viscosity affecting measurements of damping for thin forms\*.

Most of the work on seaworthiness has dealt with the vertical-plane motions of heave and pitch in head waves. Similarly the present seminar has devoted its major energies toward the hydrodynamic theories related to these same motions. However, it appears that lateral motions in a seaway, which are related to course-keeping and directional stability, are now becoming of more concern and importance as the vertical plane motions are understood to a greater degree than initially. Examining the available literature it is seen

---

\* Recently, Gerritsma, Kerwin, and Newman<sup>[43]</sup> used the same theoretical and experimental techniques to determine the damping of the conventional Series 60 forms, and in this study the agreement between theory and experiments was very much improved.

that two-dimensional section damping and virtual mass data for lateral motion and also for roll are available in the work of Grim<sup>[32]</sup> and also in some Japanese work<sup>[33]</sup>. The importance of these quantities for the case of zero forward speed is very evident, but serious questions arise as to their relative importance at speed. This is due to the occurrence of large hydrodynamic forces and moments due to angle of attack and angular rotation, which arise due to the action of hydrodynamic lift forces. These hydrodynamic forces are much larger than the damping forces due to oscillation on the free surface, even if the effect of forward speed is included in the calculation of the damping. Thus emphasis is shifted toward the dynamic forces that determine the calm water lateral dynamic stability of the ship, with possible small modification due to the frequency effects (mainly at low frequencies, which are the most important for lateral motion). An example of a recent study that makes use of some of these ideas is that by Rydill<sup>[34]</sup>, but more work will have to be carried out in order to put this approach on a more rational basis.

### 2.3 Exciting Forces on Ships in Waves

With regard to the problem of the determination of the exciting forces acting on ships in waves, the major effort to date has concerned itself with applying the results obtained for submerged bodies. The application of slender body theory to the evaluation of the forces acting on submerged bodies under waves (neglecting any free surface effects and self-wave formation) has shown good success in achieving a simple representation (see the work of Kaplan and Hu,<sup>[22]</sup>). Recent experimental studies<sup>[23]</sup> have given support to this theory, and the simple interpretation in terms of virtual mass and fluid acceleration is a useful tool in further extension of these results. Modification of the virtual mass term to account for frequency dependence, associated with free surface influence, leads to direct application to the surface ship problem, as shown by Korvin-Kroukovsky and Jacobs<sup>[1]</sup> and Kaplan<sup>[24]</sup>. The limited experimental data on the vertical force and pitching moment on a restrained ship model in regular waves, as given by Korvin-Kroukovsky and Jacobs, shows good agreement with theory, thereby giving support to this method. It is intended to extend this method to other forces, such as side force and yawing moment in oblique waves, with some partial account of diffraction effects in allowing the ship to make its own waves. Subsequent to the seminar Hu<sup>[35]</sup> has published results of this type.

As far as three-dimensional approaches are concerned, the method used by Cummins<sup>[25]</sup> to find the forces on a submerged body under waves is presently the most complete. However, a knowledge of the singularity distribution representing the body is necessary, and there does not appear to be any simple way of extending his method to the free surface. As far as the results for a submerged body are concerned however, they are the same as those obtained by the two-dimensional slender body technique of Kaplan and Hu<sup>[26]</sup>.

Subsequent to the Seminar Newman<sup>[27]</sup>, has called attention to the work of Haskind<sup>[28]</sup>, who showed that at zero speed the exciting forces due to waves can be determined exactly, including the diffraction effect, without solving the diffraction problem. In fact one need only know the wave amplitude coefficient for forced motion in the direction from which the wave is coming and at the frequency of the wave. This is a fine result and the possibility of extension to forward speed is under active consideration by a number of investigators.

## 2.4 Problems and Recommendations

### 2.4.1 Consistent and Inconsistent Linearizations

By consistent linearization we mean the systematic expansion of the full nonlinear problem in terms of one or more small parameters. Examples are the thin-ship and raft theories of Stoker and Peters, and Wehausen, the thin-ship theory of Newman and a theory for slender surface ships published recently by Vossers. Theories in which slender submerged bodies are treated assuming the linearized free surface condition ab initio can probably be made consistent if we regard them as resulting from an expansion in two small parameters, slenderness and reciprocal submergence, although the nature of the expansion is not clear. Inconsistent theories arise whenever we assume the linearized free surface condition ab initio and do not otherwise restrict the problem. Much of the work on submerged finite bodies is of this type, e.g. Havelock's work on the wave resistance of a submerged circular cylinder, MacCamy's work on the motions of shallow cylinders (rafts), strip theory based on section coefficients of full sections, attempts at computing wave resistance assuming the linear free surface conditions while satisfying the boundary conditions on the hull, etc.

It was the feeling of the participants that more problems should be considered by inconsistent methods, since it is possible thereby to get concrete results. At the same time it was suggested that the time has come to consider the simplest cases of a body (or singularity) using the exact free surface condition, or a more exact approximation to it. One approach suggested was to consider a line dipole (two-dimensional problem) numerically, as a function of the dipole strength and depth, to establish the range of strength-depth combinations in which the linearized treatment is useful. By letting the strength increase or the depth decrease one would hope to discover what characteristically nonlinear features appear. It was also proposed that this problem be treated analytically.

### 2.4.2 Relations Between Cross-Coefficients of Inertia and Damping

In the treatment of a body which is at the same time in uniform translation and undergoing monochromatic small oscillations involving six degrees of freedom two six by six matrices arise. One of these is the matrix  $B_{ij}$  of damping\*coefficients, where  $i$  denotes the degree of freedom on which the damping force acts and  $j$  denotes the degree of freedom whose velocity gives rise to the damping force. The other matrix is defined similarly but involves accelerations. Various questions arise regarding the symmetry properties of these matrices and the vanishing of certain elements thereof. The dependence of the answers on the type of linearization assumed and on the symmetry properties of the body should be determined.

After the conclusion of the seminar, Newman and Timman have looked into the question for the so-called\* damping matrix, assuming that the body has transverse and longitudinal symmetry and is sufficiently thin, slender or submerged that linear theory is valid. The results<sup>[29]</sup> are as follows:

	Surge j=1	Sway j=2	Heave j=3	Roll j=4	Pitch j=5	Yaw j=6
Surge force i=1	$B_{11}$	$B_{12}=0$	$B_{13}=-B_{31}$	$B_{14}=0$	$B_{15}=B_{51}$	$B_{16}=0$
Sway force i=2	$B_{21}=0$	$B_{22}$	$B_{23}=0$	$B_{24}=B_{42}$	$B_{25}=0$	$B_{26}=-B_{62}$
Heave force i=3	$B_{31}=-B_{13}$	$B_{32}=0$	$B_{33}$	$B_{34}=0$	$B_{35}=-B_{53}$	$B_{36}=0$
Roll moment i=4	$B_{41}=0$	$B_{42}=B_{24}$	$B_{43}=0$	$B_{44}$	$B_{45}=0$	$B_{46}=-B_{64}$
Pitch moment i=5	$B_{51}=B_{15}$	$B_{52}=0$	$B_{53}=-B_{35}$	$B_{54}=0$	$B_{55}$	$B_{56}=0$
Yaw moment i=6	$B_{61}=0$	$B_{62}=-B_{26}$	$B_{63}=0$	$B_{64}=-B_{46}$	$B_{65}=0$	$B_{66}$

\* This appellation is improper when there is coupling between two or more modes of oscillation.



Subsequently Lurye<sup>[36]</sup>, has investigated the symmetry properties of the matrix connecting the six complex forces and moments with the six corresponding complex velocities.

#### 2.4.3 Relations of the Kramers-Kronig Type Between Damping and Inertia Coefficients

At the seminar it was suggested by Kotik that the wave-amplitude coefficient and the added-mass, as functions of frequency, are related in a manner similar to the real and imaginary parts of the complex dielectric constant of a material, or the resistive and reactive parts of the impedance of a linear passive network. If the appropriate relation were found it would be very useful. For instance, it would provide an internal consistency check on added-masses and wave-amplitude coefficients computed by approximate methods.

Subsequent to the seminar the details were worked out<sup>[30]</sup> for the case of zero speed. Some numerical consequences are presented in Figures 6 and 7.

#### 2.4.4 Effect of Viscosity in Wave Problems

At the seminar it was mentioned that certain experiments performed by Gerritsma on ship forms varying in beam had led to peculiar results for thin models. Some aspects of the experimental results obtained in England on the motions of thin planks are also not predicted by theory. It was suggested that viscosity might be important for thin models, and although no conclusion was reached the following theoretical problems were proposed: to find the waves generated by a heaving vertical plate of zero thickness, and by a rolling circular cylinder. These problems are of special interest because in the absence of viscosity no waves would be produced. After the seminar Karp found that an approximate expression for the wave amplitude coefficient for a heaving vertical plate in a viscous fluid, valid when  $kl \ll 1$ , is

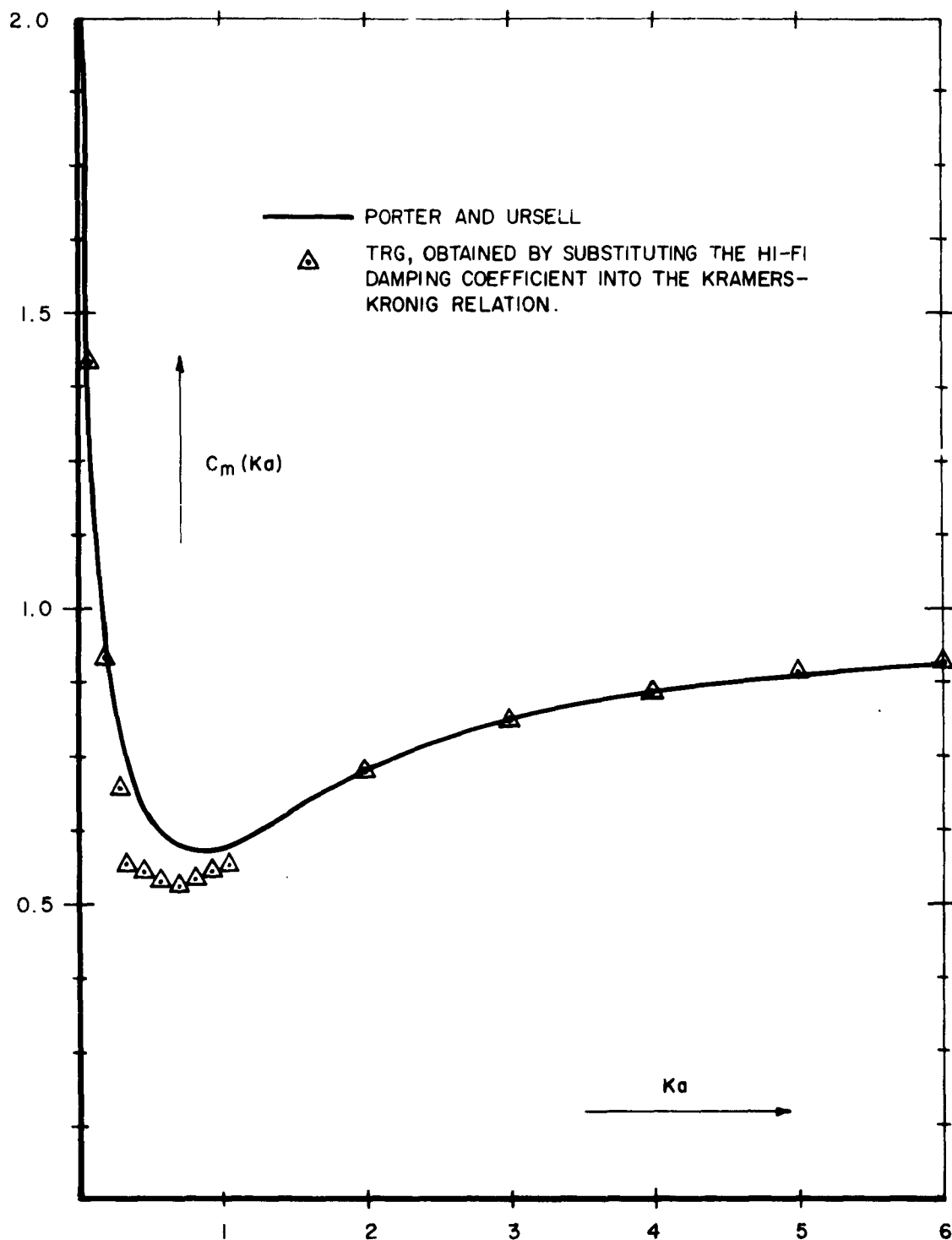


FIGURE 6  
ADDED-MASS PARAMETER FOR A  
HEAVING CIRCULAR CYLINDER

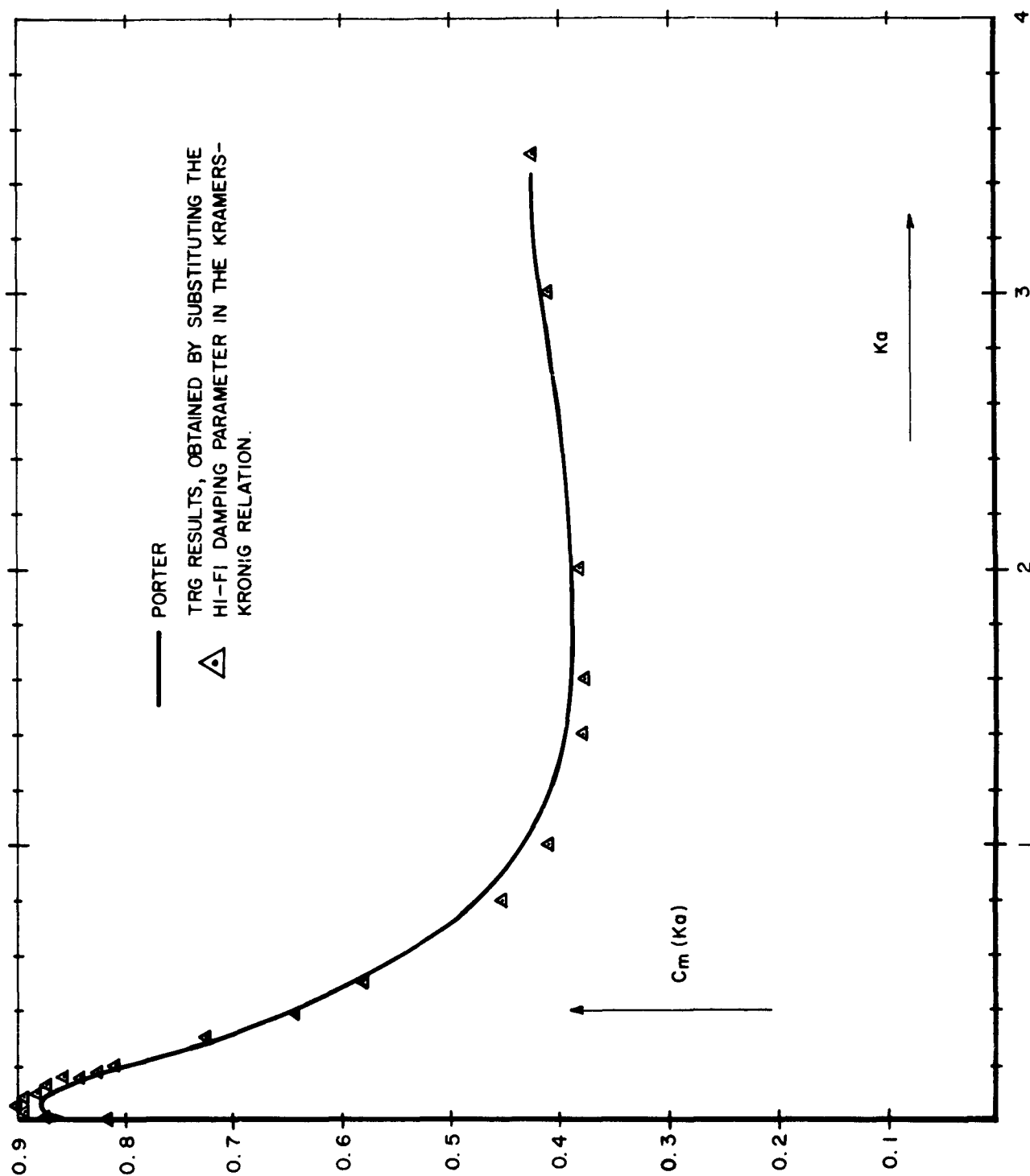


FIGURE 7  
 ADDED-MASS PARAMETER FOR A HEAVING SPHERE

$$2kl,$$

where

$$k = \sigma^2/g$$

and

$$l = \sqrt{\nu/\sigma}$$

is a measure of the boundary layer thickness in an infinite fluid.

#### 2.4.5 Time-Domain Formulation of the Equations of Motion

At the seminar the question of the form of the equations of motion for nonmonochromatic excitation was brought up, and the usual remarks regarding the frequency-dependence of the damping and added-mass coefficients were made. It was suggested that the appropriate equations would be integro-differential equations with convolution kernels. Subsequently Cummins<sup>[31]</sup> has supplied the appropriate analysis, and the questions have been largely resolved. In the linear range the time and frequency-domain formulations are completely equivalent.

#### 2.4.6 Recommendations

It appears that a number of the problems put forth at the seminar have subsequently been solved, and the seminar may be credited with a directly stimulating role in this regard. Most of the participants felt that they benefited in various ways not concretely reflected in subsequent publications but valuable none the less. It appears that a small meeting of specialists, like this seminar, can fruitfully be held at intervals of about three years.

The value of such a seminar to the field as a whole could be improved by introducing a suitable mechanism for recording the principal points brought out in discussion. The method used in this seminar was to tape record the discussion. Analysis of the records was unfortunately delayed, and the method is in any case not satisfactory. It is recommended that in the future one of the participants be assigned to write up each half-day of discussion, immediately after it occurs. The writeups should be circularized and amended during the seminar and before it concludes, and the final version should be made available for distribution shortly thereafter.

REFERENCES

- [ 0 ] Vossers, G., "Fundamentals of the Behaviour of Ships in Waves," a series of papers in "International Shipbuilding Progress" beginning with Vol. 6, November 1959.
  
- [ 1 ] Korvin-Kroukovsky, B.V., Jacobs, W.R., "Pitching and Heaving Motions of a Ship in Regular Waves", Tr. SNAME, 65, 1957, p. 590-632.
  
- [ 2 ] Weinblum, G., St. Denis, M., "On the Motion of Ships at Sea", Tr. SNAME, 58, 1950. p. 184-248.
  
- [ 3 ] Havelock, T.H., "Damping of the Heaving and Pitching Motion of a Ship", Phil. Mag. 33, p. 666-673, "Notes on the Theory of Heaving and Pitching", TINA, 37, 1945, p. 109-122.
  
- [ 4 ] Holstein, H., "Untersuchungen an einen Tauschwingungen ausfuhrenden Quader", Werft Reederei Hafen, 17, 1936, p. 385-389.
  
- [ 5 ] Grim, O., "Berechnung der durch Schwingungen eines Schiffskorpers erzeugten hydrodynamischen Krafte", J.S.T.G., 47, 1953, p. 277-299.
  
- [ 6 ] Ursell, F., "On the Heaving Motion of a Circular Cylinder on the Surface of a Fluid", Quart. J. Mechanics & Applied Math., 2, 1949, p. 218-231.
  
- [ 7 ] Porter, W.R., "Pressure Distributions, Added-Mass, and Damping Coefficients for Cylinders Oscillating in a Free Surface", University of California, July 1960, Series No. 82, Issue No. 16.
  
- [ 8 ] Kaplan, P., Jacobs, W.R., "Two-Dimensional Damping Coefficients from Thin Ship Theory", Note No. 586, Stevens Institute of Technology, Davidson Laboratory, April 1960.

- [ 9] Tasai, F., "On the Damping Force and Added Mass of Ship's Heaving and Pitching", J. Zosen Kiokai, 105, 1959, p. 47-56.
- [10] MacCamy, R.C., "On the Heaving Motion of Cylinders of Shallow Draft", Journal of Ship Research, Vol. 5, No. 3, Dec. 1961, p. 34-43.
- [11] Ursell, F., "On the Rolling Motion of Cylinders in the Surface of a Fluid", Quart. J. Mechanics & Applied Math., 2, 1949, p. 335-353.
- [12] Holstein, H., "Über die Verwendung des Energiesatzes zur Lösung von Oberflächenwellenproblemen", Ing. Arch., 8, 1937, p. 103-111.
- [13] Stelson, Thomas E., "Acceleration of Bodies in Fluids - A Study of Virtual Mass", Carnegie Institute of Technology, May 1952.
- [14] Havelock, T.H., "Waves due to a Floating Sphere Making Periodic Heaving Oscillations", PRS A231, 1955, p. 1-7.
- [15] Ursell, F., unpublished, quoted by Barakat on p. 50 of [16].
- [16] Barakat, R., "Vertical Motion of a Floating Sphere in a Sine Wave Sea - I. Radiation and Diffraction Problems", Information Technology Laboratories, (OD-60-6).
- [17] TRG, Incorporated (Unpublished).
- [18] Private Communication.
- [19] Peters, A.S. and Stoker, J.J., "A Uniqueness Theorem and A New Solution for Sommerfeld and Other Diffraction Problems", Comm. Pure Appl. Math., 1954, 7, p. 565.

- [20] Newman, J.N., "A Linearized Theory for the Motion of a Thin Ship in Regular Waves", Journal of Ship Research, June 1961, Vol. 5, No. 1, p. 34-55.
- [21] Vossers, G. [0].
- [22] Kaplan, Paul and Hu, Pung Nien, "Three-Dimensional Stripwise Damping Coefficients for Heave and Pitch of a Submerged Slender Spheroid", Journal of Ship Research, June 1960, Vol. 4, No. 1, p. 1-7.
- [23] Henry, C.J., Kaplan, P., Martin, M.M., "Wave Forces on Submerged Bodies", Stevens Institute of Tech., Davidson Lab., Rpt. 808, June 1961.
- [24] Kaplan, P., "The Application of Slender Body Theory to the Forces Acting on Submerged Bodies and Surface Ships in Regular Waves", Journal of Ship Research, Nov. 1957.
- See also -  
Kaplan, P., Hu, Pung Nien, "On the Lateral Damping Coefficients of Submerged Bodies of Revolution", Stevens Inst. of Tech., Davidson Lab., Rpt. 830, Feb. 1962.
- [25] Cummins, W.E., "The Force and Moment on a Body in a Time-Varying Potential Flow", Journal of Ship Research, April 1957, Vol. 1, No. 1, p. 7-18.
- [26] Kaplan, P., Hu, Pung Nien, "Virtual Mass and Slender-Body Theory for Bodies in Waves", Proceedings of the Sixth Annual Conference on Fluid Mechanics, University of Texas, September 1959.
- [27] Newman, J.N., "The Exciting Forces on Fixed Bodies in Waves", DTMB, June 1962.

- [28] Haskind, M.D., "Exciting Forces and Moments on Ships in Waves", Proceedings of the Academy of Science, No. 7, 1947, (USSR).
- [29] Newman, J.N., Timman, R., "The Coupled Damping Coefficients of a Symmetric Ships", Journal of Ship Research, March 1962, Vol. 5, No. 4.
- [30] Kotik, J., Mangulis, V., "On the Kramers-Kronig Relations for Ship Motions", International Shipbuilding Progress, Vol. 9, No. 97, September 1962.
- [31] Cummins, W.E., "The Impulse Response Function and Ship Motions", Symposium on Ship Theory, Institut fur Schiffbau der Universitat Hamburg, Hamburg, Germany, Jan. 25-27, 1962.
- [32] Grim, O., "Die hydrodynamischen Krafte bien Rollversuch", Schiffstechnik, 3, 1956.
- [33] "Advances in Calculation of Wavemaking Resistance of Ships," Vol. 2, 60th Anniversary Series, Society of Naval Architects of Japan, 1960.
- [34] Rydill, L.J., "A Linear Theory for the Steered Motion of Ships in Waves," Transactions, Institute of Naval Architects, 1959.
- [35] Hu, Pung Nien, "Lateral Force and Moment on Ships in Oblique Waves", Journal of Ship Research, June 1962.
- [36] Lurye, J., TRG Document P-720, July 1962.
- [37] Eggers, K., "Uber die Erfassung der Widerstandserhohung im Seegang durch Energiebetrachtungen", Ing. Archiv, 29 Bd. 1, Heft, 1960.



- [38 ] Haskind, M.D. and Riman, I.S., "A Method of Determining the Pitching and Heaving Characteristics of Ships", DTMB Translation No. 253, 1955.
- [39a] Gerritsma, J., "Experimental Determination of Damping, Added Mass and Added Mass Moment of Inertia of a Ship Model", International Shipbuilding Progress, 1957.
- [39b] Gerritsma, J., "Ship Motions in Longitudinal Waves," International Shipbuilding Progress, Vol. 7, No. 66, 1960.
- [40] Golovato, P., "A Study of the Forces and Moments on a Surface Ship Performing Heaving Oscillations", DTMB Rpt. No. 1074, August 1946, ...together with unpublished work referred to in discussion of [1].
- [41] Newman, J.N., Unpublished results.
- [42] Gerritsma, J., Private communication to J.N. Newman.
- [43] Gerritsma, J., Kerwin, J.E., and Newman, J.N., "Polynomial Representation and Damping of Series 60 Hull Forms", International Shipbuilding Progress, Vol. 9, No. 95, July 1962.

## DISTRIBUTION LIST

Contract Nonr-3175(00)

### No. of Copies

15	Commanding Officer and Director, David Taylor Model Basin Washington 7, D.C., Attention Code 513
7	Chief, Bureau of Ships, Department of the Navy, Washington 25, D.C., Technical Code 335 <ul style="list-style-type: none"><li>1 Ships Research (Code 341B)</li><li>1 Lab. Management (Code 320)</li><li>1 Ship Design (Code 410)</li><li>1 Preliminary Design (Code 420)</li><li>1 Hull Design (Code 440)</li></ul>
2	Chief of Naval Research, Department of the Navy, Washington 25, D.C., (Code 438)
1	Commanding Officer, Office of Naval Research Branch Office, 495 Summer Street, Boston 10, Massachusetts
1	Commanding Officer, Office of Naval Research Branch Office, 346 Broadway, New York 15, New York
1	Commanding Officer, Office of Naval Research Branch Office, The John Crerar Library Building, 86 East Randolph Street, Chicago 1, Illinois
1	Commanding Officer, Office of Naval Research Branch Office 1000 Geary Street, San Francisco 9, Calif.
1	Commanding Officer, Office of Naval Research Branch Office, 1030 East Green Street, Pasadena 1, Calif.
2	Director, Naval Research Laboratory, Washington 20, D.C. Attn: Technical Information Office (Code 2021)
2	Commander, Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland, Attn: Library.
2	National Aeronautics and Space Administration, 1512 H Street, N.W., Washington 25, D.C., For additional Distribution to: Langley Research Center
1	Director, Institute of Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland

No. of  
Copies

- 1 Dr. L. Landweber, Iowa Institute of Hydraulic Research,  
State University of Iowa, Iowa City, Iowa.
- 1 Director, St. Anthony Falls Hydraulic Laboratory, University  
of Minnesota, Minneapolis, Minnesota
- 1 Head, Department of Naval Architecture and Marine Engineering,  
Massachusetts Institute of Technology, Cambridge 39, Mass.
- 1 Editor, Engineering Index, Engineering Societies Library,  
29 West 39th Street, New York 18, New York.
- 1 Director, Hydrodynamics Laboratory, California Institute of  
Technology, Pasadena 4, Calif.
- 1 Head, Department of Civil Engineering, Colorado State University  
Fort Collins, Colorado
- 1 Prof. G. Birkhoff, Department of Mathematics, Harvard University  
Cambridge 38, Mass.
- 1 Dr. J.V. Wehausen, Institute of Engineering Research, University  
of California, Berkeley 4, Calif.
- 1 Prof. M.A. Abkowitz, Department of Naval Architecture and  
Marine Engineering, Massachusetts Institute of Technology,  
Cambridge 39, Mass.
- 1 Dr. David Gilbarg, Department of Mathematics, Indiana University,  
Bloomington, Indiana
- 1 Prof. F.M. Lewis, Department of Naval Architecture and Marine  
Engineering, Massachusetts Institute of Technology,  
Cambridge 39, Massachusetts
- 4 Davidson Laboratory, Stevens Institute of Technology,  
711 Hudson Street, Hoboken, New Jersey;
  - 2 Dr. J.P. Breslin
  - 1 Prof. Korvin-Kroukovsky
  - 1 Mr. L. Ward
- 2 Institute for Mathematical Sciences, New York University,  
25 Waverly Place, New York 3, New York;
  - 1 Dr. A.S. Peters
  - 1 Prof. J.J. Stoker
- 1 Dr. H.L. Pond, 4 Constitution Road, Lexington, Mass.

**No. of  
Copies**

- 1 Hydronautics, Incorporated, 200 Monroe Street, Rockville, Md.
- 1 Dr. M. St Denis - Weapons System Evaluation Group,  
Institute for Defense Analysis, Room 1-E-862, The Pentagon,  
Washington 25, D.C.
- 1 California Institute of Technology, Hydrodynamics Laboratory,  
Pasadena, California
- 1 Editor, Applied Mechanics Reviews, Southwest Research Institute,  
8500 Culebra Road, San Antonio 6, Texas
- 1 Librarian, Society of Naval Architects and Marine Engineers,  
74 Trinity Place, New York 6, New York